Math 72: Selected Review Chapters 7-8

Solve the equations for all complex solutions.

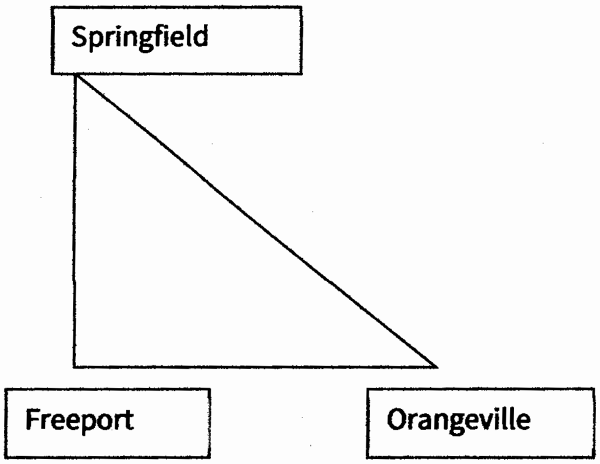
1) $\sqrt{2x-5}=3$
2) $x^2=9$
3) $(2x-5)^{1/2}=-3$
4) $\sqrt{x+11}+3=x+2$
5) $x^2=3$
6) $(2x-5)^{1/3}=-3$
7) $\sqrt{2x-5}=-3$
8) $x^2=-3$
9) $4x^3-3=29$
10) $\sqrt{x+2}-\sqrt{x+9}=7$
11) $2x^2+4x-6=0$
12) $(2x-5)^{2/3}=3$
13) $\sqrt[3]{2x-5}=3$
14) $3x^2+2x-4=0$
15) $2x^{2/3}+4x^{1/3}-6=0$
16) $\sqrt[4]{2x-5}=3$
17) $3(2x-4)^2=-6$
18) $3x^2+2x+4=0$

Find all intercept(s) of the function.

19) $f(x)=2x^2+4x-6$
20) $f(x)=3x^2+2x-4$
21) $f(x)=3x^2+2x+4$

22) Simplify $\left(\frac{16x^{1/3}}{x^{-1/3}}\right)^{-1/2}+\left(\frac{x^{-3/2}}{64x^{-1/2}}\right)^{1/3}$

23) Because of the increase in traffic between Springfield and Orangeville, a new road was built to connect the two towns. The old road goes south x miles from Springfield to Freeport and then goes east $x+3$ miles from Freeport to Orangeville. The new road is 9 miles long and goes straight from Springfield to Orangeville. Write and solve an algebraic equation to find the number of miles that a person saves by driving the new road instead of the old road. Give an exact answer and then round to the nearest tenth.



Math 72 Selected Review Chapters 7-8

Solve the equations for all complex solutions.

* Recall *

Any real number can be written as a complex number by adding $+0i$. This instruction includes real solutions.

① $\sqrt{2x-5} = 3$

$(\sqrt{2x-5})^2 = 3^2$ square both sides

$2x - 5 = 9$

$2x = 14$

$\boxed{x = 7}$ check for extraneous

$\sqrt{2 \cdot 7 - 5} = 3$ ✓

② $x^2 = 9$

$x = \pm\sqrt{9}$ square root property

$\boxed{x = 3, -3}$

③ $(2x-5)^{1/2} = -3$

$\sqrt{2x-5} = -3$

principal square root

cannot be negative

$\boxed{\text{no solution}}$

④ $\sqrt{x+11} + 3 = x + 2$

$\sqrt{x+11} = x - 1$ isolate $\sqrt{\ }$

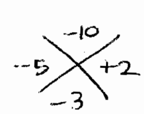
$(\sqrt{x+11})^2 = (x-1)^2$ square both sides, using ( )

$x + 11 = (x-1)(x-1)$ FOIL

$x + 11 = x^2 - 2x + 1$ Recognize quadratic,

$0 = x^2 - 3x - 10$ set $= 0$

$0 = (x-5)(x+2)$



$\boxed{x = 5}$ $x \cancel{=} -2$

check for extraneous

$\sqrt{5+11} + 3 = 5 + 2$ ✓ $\sqrt{-2+11} + 3 = -2 + 2$ No.

(5) $x^2 = 3$

$\boxed{x = \pm\sqrt{3}}$ square root property

or $\boxed{x = \sqrt{3}, -\sqrt{3}}$

(6) $(2x-5)^{1/3} = -3$

$\sqrt[3]{2x-5} = -3$ cube roots <u>can</u> be negative! (odd index)

$(\sqrt[3]{2x-5})^3 = (-3)^3$

$2x-5 = -27$

$2x = -22$

$\boxed{x = -11}$

(7) $\sqrt{2x-5} = -3$

same as (3)

(8) $x^2 = -3$

$x = \pm\sqrt{-3}$ square root property

$\boxed{x = \pm i\sqrt{3}}$

or $\boxed{x = i\sqrt{3}, -i\sqrt{3}}$

(9) $4x^3 - 3 = 29$ isolate cube

$4x^3 = 32$

$x^3 = 8$ cubes!

$x^3 - 8 = 0$ difference of cubes can be factored

$(x-2)(x^2+2x+4) = 0$ set factors $= 0$.

$x - 2 = 0$ $\quad x^2 + 2x + 4 = 0$

$\boxed{x = 2}$ $\quad x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{4-16}}{2}$

$= \dfrac{-2 \pm \sqrt{-12}}{2} = \dfrac{-2}{2} \pm \dfrac{2i\sqrt{3}}{2}$

$\boxed{x = -1 + i\sqrt{3}, \ -1 - i\sqrt{3}}$

(10) $\sqrt{x+2} - \sqrt{x-9} = 7$ isolate one square root

$(\sqrt{x+2})^2 = (\sqrt{x-9} + 7)^2$ square both sides, using ( )

$x+2 = (\sqrt{x-9}+7)(\sqrt{x-9}+7)$

$x+2 = x-9 + 7\sqrt{x-9} + 7\sqrt{x-9} + 49$

$x+2 = x + 14\sqrt{x-9} + 40$ isolate other square root

$-38 = 14\sqrt{x-9}$

$\frac{-38}{14} = \sqrt{x-9}$

principal square root cannot be negative.

$\boxed{\text{no Solution}}$

(11) $\frac{2x^2}{2} + \frac{4x}{2} - \frac{6}{2} = \frac{0}{2}$ divide both sides by 2

$x^2 + 2x - 3 = 0$ factor

$(x+3)(x-1) = 0$

$\boxed{x = -3 \quad x = 1}$

(12) $(2x-5)^{2/3} = 3$

$(\sqrt[3]{2x-5})^2 = 3$ square root property

$\sqrt[3]{2x-5} = \pm\sqrt{3}$ write two equations

$(\sqrt[3]{2x-5})^3 = (\sqrt{3})^3$ $\quad (\sqrt[3]{2x-5})^3 = (-\sqrt{3})^3$ cube both sides

$2x-5 = \sqrt{27}$ $\quad 2x-5 = -\sqrt{27}$ $\quad \sqrt{3}\cdot\sqrt{3}\cdot\sqrt{3} = \sqrt{27}$

$2x-5 = 3\sqrt{3}$ $\quad 2x-5 = -3\sqrt{3}$ simplify radical

$2x = 5+3\sqrt{3}$ $\quad 2x = 5-3\sqrt{3}$

$\boxed{x = \frac{5+3\sqrt{3}}{2}, \frac{5-3\sqrt{3}}{2}}$

(13) $\sqrt[3]{2x-5} = 3$ cube both sides

$(\sqrt[3]{2x-5})^3 = 3^3$

$2x-5 = 27$

$2x = 32$

$\boxed{x = 16}$

(14) $3x^2 + 2x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(3)(-4)}}{2(3)}$$

$$= \frac{-2 \pm \sqrt{52}}{6}$$

$$= \frac{-2}{6} \pm \frac{2\sqrt{13}}{6}$$

$$\boxed{x = \frac{-1 \pm \sqrt{13}}{3}}$$

or $\boxed{x = \frac{-1+\sqrt{13}}{3},\ \frac{-1-\sqrt{13}}{3}}$

$b^2 - 4ac$
$2^2 - 4(3)(-4)$
$4 + 48$
52 not a perfect square, does not factor

52
2 26
2 13
$52 = 2^2 \cdot 13$

(15) $\frac{2x^{2/3}}{2} + \frac{4x^{1/3}}{2} - \frac{6}{2} = \frac{0}{2}$ divide all by 2

$x^{2/3} + 2x^{1/3} - 3 = 0$ substitute $u = x^{1/3}$

$u^2 + 2u - 3 = 0$

$(u+3)(u-1) = 0$

-3
3 -1
2

$u = -3 \qquad u = 1$ replace u by $x^{1/3}$

$x^{1/3} = -3 \qquad x^{1/3} = 1$

$\sqrt[3]{x} = -3 \qquad \sqrt[3]{x} = 1$ cube both sides

$(\sqrt[3]{x})^3 = (-3)^3 \qquad (\sqrt[3]{x})^3 = 1^3$

$\boxed{x = -27} \qquad \boxed{x = 1}$

(16) $\sqrt[4]{2x-5} = 3$ raise to 4th power

$(\sqrt[4]{2x-5})^4 = 3^4$

$2x - 5 = 81$

$2x = 86$

$\boxed{x = 43}$ check for extraneous

$\sqrt[4]{2 \cdot 43 - 5} = 3$

$\sqrt[4]{81} = 3$ ✓

(17) $3(2x-4)^2 = -6$ isolate square

$(2x-4)^2 = -2$ square root property

$2x-4 = \pm\sqrt{-2}$ simplify imaginary number

$2x-4 = \pm i\sqrt{2}$

$2x = 4 \pm i\sqrt{2}$

$x = \frac{4}{2} \pm \frac{i\sqrt{2}}{2}$

$\boxed{x = 2 \pm \frac{i\sqrt{2}}{2}}$

OR $\boxed{x = 2 + \frac{i\sqrt{2}}{2},\ 2 - \frac{i\sqrt{2}}{2}}$

(18) $3x^2 + 2x + 4 = 0$

$b^2 - 4ac$

$2^2 - 4(3)(4)$

$4 - 48$

-44 not positive perfect square

not factorable

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(4)}}{2(3)}$

$= \frac{-2 \pm \sqrt{-44}}{6}$

$= \frac{-2}{6} \pm \frac{2i\sqrt{11}}{6}$

44

2 ^ 22

2 ^ 11

$\boxed{x = -\frac{1}{3} \pm \frac{i\sqrt{11}}{3}}$

OR $\boxed{x = -\frac{1}{3} + \frac{i\sqrt{11}}{3},\ -\frac{1}{3} - \frac{i\sqrt{11}}{3}}$

Find all intercepts means both x and y intercepts.

x-intercept: set $y=0$

y-intercept: set $x=0$.

(19) y-int $f(0) = 2(0)^2 + 4(0) - 6 = -6$ $\boxed{(0,-6) \text{ y-int}}$

x-int $0 = 2x^2 + 4x - 6$

see (11)

$\boxed{(-3,0) \text{ and } (1,0) \text{ x-ints}}$

(20) y-int $f(0) = 3(0)^2 + 2(0) - 4 = -4$ $\boxed{(0,-4) \text{ y-int}}$

x-int $0 = 3x^2 + 2x - 4$

see (14) $\boxed{\left(\frac{-1+\sqrt{13}}{3}, 0\right) \quad \left(\frac{-1-\sqrt{13}}{3}, 0\right) \text{ x-ints}}$

(21) y-int $f(0) = 3(0)^2 + 2(0) + 4 = 4$ $\boxed{(0,4) \text{ y-int}}$

x int $0 = 3x^2 + 2x + 4$

see (18) imaginary parts! $\boxed{\text{no x-ints}}$

(22) Simplify

$$\left(\frac{16x^{1/3}}{x^{-1/3}}\right)^{-1/2} + \left(\frac{x^{-3/2}}{64x^{-1/2}}\right)^{1/3}$$

neg exp outside means reciprocal

$$= \left(\frac{x^{-1/3}}{16x^{1/3}}\right)^{1/2} + \left(\frac{x^{-3/2}}{64x^{-1/2}}\right)^{1/3}$$

simplify inside ( )

neg exp inside means move to other part of fraction (3 neg exponents)

$$= \left(\frac{1}{16\underbrace{x^{1/3}x^{1/3}}_{\text{add exp}}}\right)^{1/2} + \left(\frac{x^{1/2}}{64x^{3/2}}\right)^{1/3}$$

subtract exponents $\frac{3}{2} - \frac{1}{2} = 1$

$$= \left(\frac{1}{16x^{2/3}}\right)^{1/2} + \left(\frac{1}{64x}\right)^{1/3}$$

apply outer exponent

$$= \frac{1^{1/2}}{16^{1/2} \cdot (x^{2/3})^{1/2}} + \frac{1^{1/3}}{64^{1/3} \cdot x^{1/3}}$$

radical mult exp radical

$\frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{2}} = \frac{1}{3}$

$$= \frac{1}{\sqrt{16}\, x^{1/3}} + \frac{1}{\sqrt[3]{64}\, x^{1/3}}$$

$\sqrt{16} = 4$

$\sqrt[3]{64} = 4$

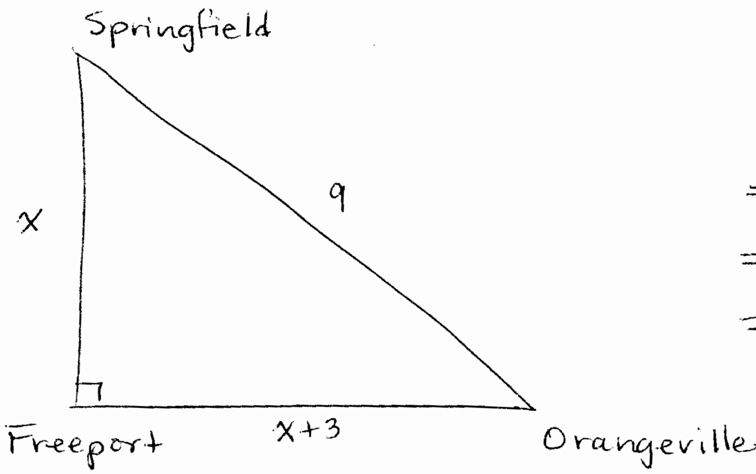
$$= \frac{1}{4x^{1/3}} + \frac{1}{4x^{1/3}}$$

combine like terms

$$= \frac{2}{4x^{1/3}} = \boxed{\frac{1}{2x^{1/3}}}$$

reduce

23



miles saved

= old route − new route

$= x + x + 3 - 9$

$= 2x - 6$. But we need x!

Pythagorean Theorem $a^2 + b^2 = c^2$

$$x^2 + (x+3)^2 = 9^2$$

FOIL

$$x^2 + (x+3)(x+3) = 81$$

$$x^2 + x^2 + 6x + 9 = 81$$

set = 0

$$\frac{2x^2}{2} + \frac{6x}{2} - \frac{72}{2} = \frac{0}{2}$$

divide both sides by 2

$$x^2 + 3x - 36 = 0$$

$b^2 - 4ac$

$3^2 - 4(1)(-36)$

$9 + 144$

153 not a perfect square, does not factor

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-36)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9 + 144}}{2}$$

$$= \frac{-3 \pm \sqrt{153}}{2}$$

or $\frac{-3 + \sqrt{153}}{2}$ $\cancel{\frac{-3 - \sqrt{153}}{2}}$ = negative extraneous

saved $2x - 6$

$$= \cancel{2}\left(\frac{-3 + \sqrt{153}}{\cancel{2}}\right) - 6$$

$$= \boxed{\sqrt{153} - 9 \text{ miles}}$$

≈ 3.369

$\boxed{\approx 3.4 \text{ miles}}$